D 40042

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Name.....................................

Reg. No.................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2018

(CUCBCSS—UG)

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. What is the period of $f(z) = e^{2iz}$?
2. Give an example of an entire function.
3. What is the complex, form of Cauchy-Riemann equations ?
4. Define entire function.
5. What are the singularities of $f(z) = |z|^2$.
6. State Cauchy's integral formula.
7. State Morera's theorem.
8. State Gausss mean value theorem.
9. Define Residue of a complex function.
10. Give an example of an essential singularity.
11. Define removable singularity of a complex function.
12. What, is the residue at a removable singularity ?

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of* **fourteen** *questions.*
Each question carries 4 marks.

13. Define Analytic functions. Give an example.
14. Show that $(z) = \sin x \cos h\, y + i \cos x \sin h\, y$ is an entire function.

Turn over

15. If $f = u + iv$ is analytic, then show that u and v are harmonic.

16. Prove of disprove : $\mathrm{Log}(a^b) = b\mathrm{Log}(a)$, where Log is the principal branch of logarithm and $a, b \in \mathbb{C}$.

17. State Cauchy's integral formula and its extension.

18. Find all the values of $\sin^{-1}(-i)$.

19. Is Cauchy-Goursat theorem valid for arbitrary connected domains? Prove your claim.

20. Using Liouville's theorem prove the fundamental theorem of algebra.

21. Evaluate $\int_C \frac{e^{-z}\,dz}{z - i\pi/2}$, where C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$.

22. Suppose $z_n = x_n + iy_n, n = 1, 2, 3, \ldots\ldots$, and $S = X + iY$. If $\sum_{n=1}^{\infty} z_n = S$, then show that $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$.

23. State Laurent theorem.

24. Give an example of a non isolated singularity.

25. Using Cauchy's integral theorem, evaluate $\int_C \frac{z+1}{z^2 - 2z}\,dz$, where C is the circle $|z| = 3$ in the positive sense.

26. Define pole and its order of a complex function.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of* **nine** *questions.*
Each question carries 7 marks.

27. Derive Cauchy-Riemann equations.

28. If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then show that u and v are harmonic in D.

29. Find the harmonic conjugate of $u(x, y) = \sin h\, x \sin y$.

30. Using contour integration, evaluate $\int_C z^{1/2}\, dz$, where C is the path given by $z = 3e^{i\theta}, 0 \leq \theta \leq \pi$.

31. State and prove the principle of domination of paths.

32. Find the Laurent series that represents the function $f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$ in the domain $0 < |z| < \infty$.

33. If a power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ converges when $z = z_1 (z_1 \neq z_0)$ then show that it is absolutely convergent at each point z in the open disk $|z - z_0| < R_1$, where $R_1 = |z_1 - z_0|$.

34. State and prove Cauchy's residue theorem.

35. Using residue evaluate $\int_0^{\infty} \frac{dx}{x^4 + 1}$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of* **three** *questions.*
Each question carries 13 *marks.*

36. State and prove reflection principle.

37. (a) State and prove Liouville's Theorem.

(b) Using Liouville's theorem, prove fundamental theorem of algebra.

38. (a) Show that the power series $S(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$ is analytic each point, z interior to the circle of convergence of that series.

(b) Find the residue of $\frac{1}{z + z^2}$ at $z = 0$.

(2 × 13 = 26 marks)