C 40395

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 10—COMPLEX ANALYSIS

Time : Three Hours

Maximum : 30 Weightage

Section A

Answer **all** *the questions.*

1. What is the value of :

$$\lim_{z \to \infty}\left(\frac{2z+i}{z+1}\right).$$

2. Find the imaginary part of $z + i$.

3. $f(z) = u(x,y) + iV(x,y)$ is analytic in a domain D if and only, V is ——————— of u.

4. What is the real part of e^z ?

5. What is the period of $\sin z$?

6. Express $\cos x$ in terms of e^{ix}.

7. The value of $e^{i\pi}$ is :

   (a) 1.

   (b) e.

   (c) −1.

   (d) 0.

8. The value of $\int_{|z|=1} \frac{dz}{z-4}$ is ——————.

   (a) $2\pi i$.

   (b) 0.

   (c) 2π.

   (d) 4.

9. The region of convergence of the series :

   $$1 + \frac{z}{1!} + \frac{z^2}{2!} + \ldots + \frac{z^n}{n!} + \ldots \text{ is } \text{——————}.$$

10. What is the sum function of the series $1 + z + z^2 + \ldots + z^{n-1} + \ldots$

11. For $f(z) = \frac{z^2 - 4}{z - 2}$, $z = 2$ is a ——————

    (a) Removable singular point.

    (b) Pole of order 1.

    (c) Pole of order 2.

    (d) Essential singular point.

Turn over

12. Identify the poles of :

$$\frac{2z}{z^2 - 1}.$$

<div align="right">(12 × ¼ = 3 weightage)</div>

Section B

Answer all **nine** *questions.*

13. Show that $f'(z)$ does not exist at any point for $f(z) = 2x + ixy^2$.

14. If $f(z)$ and $\overline{f(z)}$ are both analytic in a domain D, prove that $f(z)$ is a constant throughout D.

15. Show that :

$$\text{Log}(1 - i) = \frac{1}{2}\ln 2 - \frac{\pi}{4}i.$$

16. Show that :

$$|\sinh z|^2 = \sinh^2 x + \sin^2 y.$$

17. State Cauchy's integral formula.

18. Evaluate :

$$\int_C \frac{dz}{z - 1}, \text{ where C is } |z - 1| = 2.$$

19. Show that when $z \neq 0$

$$\frac{e^z}{z^2} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \dots$$

20. State Cauchy's residue theorem.

21. For the function $f(z) = \frac{1 - \cosh z}{z^3}$ determine the order of the pole at $z = 0$ and the corresponding residue.

<div align="right">(9 × 1 = 9 weightage)</div>

Section C

Answer any **five** *questions.*

22. Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic and find a harmonic conjugate $v(x, y)$ of u.

23. If $f(z) = u(x, y) + iV(x, y)$ is analytic in a domain D, prove that u and V are harmonic in D.

24. Find the general solution of the equation :

$$\cosh z = \frac{1}{2}.$$

25. Evaluate $\int_C \dfrac{zdz}{\left(9 - z^2\right)\left(z + i\right)}$, where C is the circle $|z| = 2$.

26. State and prove Liouville's theorem.

27. State and prove Taylor's theorem.

28. Evaluate $\int\limits_0^\infty \dfrac{dx}{x^2 + 1}$.

(5 × 2 = 10 weightage)

Section D

Answer any **two** *questions.*

29. State and prpove maximum modulus principle.

30. Give two Laurent series expansions in powers of z for the function :

$$f(z) = \frac{1}{z\left(1 + z^2\right)}$$

the regions of validity of expansions.

31. Using residues, evaluate :

$$\int\limits_{-\infty}^{\infty} \frac{\cos x\, dx}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)}\ (a > b > 0)$$

(2 × 4 = 8 weightage)