C 60053

(Pages : 4) Name_____

Reg. No._____

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2019

(CUCBCSS)

Mathematics

MAT 6B 13 (E 02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Define a convex set.
2. What is a degenerate solution of an L.P.P. ?
3. What is a slack variable ?
4. Write the names of any two methods to a solve a transportation problem.
5. Write the following L.P.P. in standard form :

   Maximize $Z = 2x_1 - 8x_2$

   subject to $x_1 + x_2 \geq -1$

   $x_1 - x_2 \leq 3$

   $x_1, x_2 \geq 0.$
6. Show that $x_1 = 2, x_2 = 1$ is a feasible solution of the L.P.P. given below :

   Maximize $Z = 4x_1 + x_2$

   $x_1 + x_2 \leq 3$

   $x_1 - x_2 > 1$

   $x_1, x_2 \geq 0.$
7. Explain, why we not use 'Transportation Algorithm' to solve 'An assignment problem'.
8. Find the number of possible feasible solutions of the following L.P.P. :

   Maximize $Z = x_1 + x_2$

   subject to the constraints $x_1 + x_2 + x_3 \leq 5$

   $x_1, x_2, x_3 \geq 0.$
9. Write the necessary and sufficient condition for a basic feasible solution to a L.P.P. to be an optimum (maximum).
10. Write the following L.P.P. in matrix form :

    Minimize $Z = x_1 + x_2 - x_3$

    subject to $x_1 + x_3 \geq 2$

    $x_1 - x_2 \geq 4$

    $x_1, x_2 \geq 0.$

Turn over

11. Write the dual of the following L.P.P. :

Minimize $Z = x_1 + x_2$

subject to $x_1 + x_2 \geq 1$

$x_1 - x_2 \leq 1$

$x_1, x_2 \geq 0.$

12. When we say that a 'transportation problem' is unbalanced ?

(12 × 1 = 12 marks)

Section B

Answer any **nine** *out of twelve questions.*
Each question carries 2 marks.

13. Define a hyper sphere in $\mathbb{R}^n$.

14. Show that the following L.P.P. has no solution :

Maximize $Z = x_1 + x_2$

where $x_1 - x_2 \geq 0$

$3x_1 - x_2 \leq -3$

$x_1, x_2 \geq 0.$

15. Show that the intersection of two convex set is also a convex set.

16. Write a short note on 'The North-West Corner Rule'.

17. Write a short note on 'The Assignment Problem'.

18. Form the Mathematical formulation of the problem given below :

Prabha goes to the market to purchase buttons. She needs atleast 20 large buttons and 30 small buttons respectively. The shopkeeper sells buttons in two tons—(i) boxes ; and (ii) cards. A box contains 10 large buttons and 5 small buttons respectively ; whereas a card contains 2 large buttons and 5 small buttons respectively.

Determine the most economical way in which Prabha should purchase the buttons, if a box costs Rs. 25 and a card costs Rs. 10 only.

19. Write the dual problem of the following L.P.P. ;

Maximize $f(x) = 2x_1 + 5x_2 + 6x_3$

subject to the constraints $5x_1 + 6x_2 - x_3 \leq 3$

$-2x_1 + x_2 + 4x_3 \leq 4$

$x_1 - 5x_2 + 3x_3 \leq 1$

$-3x_1 - 3x_2 + 7x_3 \leq 6$

$x_1, x_2, x_3 \geq 0.$

20. Find any basic feasible solution of the following transportation problem :

	m_1	m_2	m_3	m_4	
w_1	1	2	3	4	2
w_2	4	3	2	1	2
w_3	2	1	4	3	3
	3	2	1	1	

21. Given that $a_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, b_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, c_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ can be written in the form $a_1 = \lambda_1 b_1 + \lambda_2 c_1$. Find λ_1 and λ_2.

22. Write the Mathematical formulation of an assignment problem.

23. Check whether the set $A = \{(x_1, x_2) / x_1, x_2 \in \mathbb{R} \text{ s } x_1^2 + x_2^2 = 1\}$ is a convex set.

24. Show that the vectors $a_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, c_1 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ are linearly dependent.

(9 × 2 = 18 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 5 marks.

25. Maximize $Z = 2x_1 + x_2$

where $x_1 + x_2 \leq 4$
$x_1 + 2x_2 \leq 6$
$x_1 \leq 3$
$x_1, x_2 \geq 0.$

26. Prove that a hyperplane is a convex set.

27. Prove that the set of all feasible solutions to a L.P.P. constitutes a convex set.

28. Use simplex method to solve the following L.P.P. :

Maximize $Z = 2.5x_1 + x_2$

subject to the constraints $3x_1 + 5x_2 \leq 15$
$5x_1 + 2x_2 \leq 10$
$x_1, x_2 \geq 0.$

29. Find a basic feasible solution of the following transportation problem by using Vogel's approximation method :

	I	II	III	IV	
A	5	1	3	3	34
B	3	3	5	4	15
C	6	4	4	3	12
D	4	−1	4	2	19
	21	25	17	17	80

30. Consider the problem of assigning five jobs to five persons. The assignment cost are given below :

		Job				
		1	2	3	4	5
Person	A	8	4	2	6	1
	B	0	9	5	5	4
	C	3	8	9	2	6
	D	4	3	1	0	3
	E	9	5	8	9	5

Determine the optimum assignment schedule.

31. Show that the following L.P.P. has an unbounded solution :

Maximize $Z = 4x_1 + 5x_2$

subject to
$$x_1 + x_2 \geq 1$$
$$-2x_1 + x_2 \leq 1$$
$$4x_1 - 2x_2 \leq 1$$
$$x_1, x_2 \geq 0.$$

32. The column vector [1, 1, 1] is a feasible solution to the system of equations :

$$x_1 + x_2 + 2x_3 = 4$$
$$2x_1 - x_2 + x_3 = 2.$$

Reduce the given solution to a basic feasible solution.

33. State and prove 'Minimax Theorem'.

(6 × 5 = 30 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 10 *marks.*

34. S.T. any convex combination of K different optimum solutions to a L.P.P. is again an optimum solution to the problem.

35. Use simplex method to solve :

Maximize $Z = 107x_1 + x_2 + 2x_3$

subject to the constraints
$$14x_1 + x_2 - x_3 + 3x_4 = 7$$
$$16x_1 + \frac{1}{2}x_2 - 2x_3 \leq 3$$
$$3x_1 \geq 0$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

36. Prove that a hyperplane is a closed set.