5709 (Pages : 4) Name..................................

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SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2012

(CCSS)

Mathematics—(Elective Course)

MM 6B 13 (E02)—LINEAR PROGRAMMING AND GAME THEORY

Three Hours Maximum : 30 Weightage

Part I

Answer **all** *questions.*

Maximize $Z = x_1^2 + x_2^2$, subject to $x_1 - x_3 = 3$ and $x_2 \leq 2$ is a ———.

(a) Linear Programming problem.

(b) Quadratic programming problem.

(c) Transportation problem.

(d) Assignment problem.

Define a convex set.

What is surplus variable ?

Which of the following is not a convex set in R^2 ?

(a) $\{(x, y) / x + 2y = 3\}$. (b) $\{(x, y) / x^2 + y^2 \leq 1\}$.

(c) $\{(x, y) / a < x < b\}$. (d) $\{(x, y) / x^2 + y^2 = 1\}$.

Are the vectors $\overline{a} = (1, 2, 3)$, $\overline{b} = (-6, 0, 2)$ are Orthogonal ?

Which of the following sets form a basis of R^2 ?

(a) $\{(2, 0)(3, 0)\}$. (b) $\{(0, -1)(0, 1)\}$.

(c) $\{(2, 0)(0, 2)\}$. (d) $\{(0, 0)(0, -2)\}$.

Define support of a set in E^n.

Find the convex null of the set $\{(x, y) \in R^2 / x^2 + y^2 = 3\}$.

Find a basic solution of the following system with x_3 as a non basic variable $2x_1 - x_2 + 3x_3 = 3$;
$x_1 + 2x_2 - x_3 = 4$.

Define a saddle point of a two person zero sum game.

Turn over

11. Find the dual of Maximize $Z = 3x_1 + x_2$, $2x_1 + 3x_2 \geq 5$; $x_1 + x_2 \geq 3$, $x_1 \geq 0$, $x_2 \geq 0$.

12. Express (–1, 2) as a linear combination of $(2, 0)$ and $(0, 2)$.

(12 × ¼ = 3 weig

Part II

Answer **all** *questions.*

13. Find the convex null of the set $\{(1, 2), (2, 3)\}$.

14. Show that $\bar{a} = (1, 2, 1)$; $\bar{b} = (2, 3, 0)$; $\bar{c} = (1, 2, 2)$ are linearly independent in E^3.

15. Prove that every hyperplane in R^n is convex.

16. Find a basic solution of the system

$$x_1 + 2x_2 - x_3 + x_4 = 4$$
$$x_1 - x_2 + 2x_3 - x_4 = -2$$

17. Transform the following into Standard form :

Maximize $Z = 2x_1 + 3x_3$

subject to $x_1 + x_2 \leq 1$

$3x_1 + x_2 \leq 4$

$x_1 \geq 0$; $x_2 \geq 0$

18. Convert the following into a maximization problem :

Minimize $Z = 4x_1 + 3x_2$

subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$x_1 \geq 0$; $x_2 \geq 0$

19. Obtain the dual of Maximize $Z = x_1 - x_2 + 3x_3$, subject to $x_1 + x_2 + x_3 \leq 10$; $2x_1 - x$ $2x_1 - 2x_2 + 3x_3 \leq 6$.

20. Define a loop in a transportation problem.

21. Define maximin principle in a two person zero sum game.

(9 × 1 = 9 weig

Part III

Answer any **five** *questions.*

Draw the feasible space of the following in equations :

$x_1 + 2x_2 \le 7;\ x_1 - x_2 \le 4;\ x_1 \ge 0;\ x_2 \ge 0.$

Show that $X = \{(x_1, x_2) / x_1 - 2x_2 = 2\}$ is a convex set in E^2.

Show that set of all feasible solutions of a system of equations AX $=b$ is closed convex set.

Given the system :

$2x_1 - x_2 + 2x_3 = 10,\ x_1 + 4x_2 = 18$ and $x_1, x_2 \ge 0$. Obtain a basic feasible solution starting from $(2, 4, 5)$.

Using north-west corner rule find an initial basic feasible solution of the transportation problem.

	D_1	D_2	D_3	
Q_1	3	8	7	10
Q_2	6	5	8	5
	6	5	4	

Solve the following 2×2 game

Player B

$$\text{Player A} \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}$$

Show that $(1, 2, -1)$, $(0, 1, 1)$ and $(1, 1, 1)$ generate the vector space R^3.

($5 \times 2 = 10$ weightage)

Part IV

Answer any **two** *questions.*

Use Simplex method to solve :

$$\begin{aligned} \text{Minimize } Z &= x_1 - 3x_2 + 2x_3 \\ \text{subject to } \quad 3x_1 - x_2 + 2x_3 &\le 7 \\ -2x_1 + 4x_2 &\le 12 \\ -4x_1 + 3x_2 + 8x_3 &\le 10 \\ x_1, x_2, x_3 &\ge 0. \end{aligned}$$

30. Solve the transportation problem :

	D_1	D_2	D_3	Availability
S_1	5	1	8	12
S_2	2	4	0	14
S_3	3	6	7	4
Requirement	9	10	11	

31. Solve the following game :

$$\begin{bmatrix} 1 & -3 & 2 \\ -4 & 4 & -2 \end{bmatrix}.$$

(4 × 2 = 8 weig