25707 (Pages : 3) Name..................................

Reg. No..................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2012

(CCSS)

Mathematics—Core Course

MM 6B 12—NUMBER THEORY AND LINEAR ALGEBRA

e : Three Hours Maximum : 30 Weightage

I. Answer all *twelve* questions.

1. A system of m homogeneous linear equations AX = 0 is unknown has only trivial solution if :

   (a) $m = n$. (b) $m \neq n$.

   (c) rank $(A) = m$. (d) rank $(A) = n$.

2. If the number of variables in a non–homogeneous system AX = B is n then the system possesses a unique solution if :

   $\rho[A, B] = \rho(A) =$ ———.

3. If A is a matrix of order $m \times n$ and R is a non–singular matrix of order m then $\rho(RA) =$ ———.

4. If $n > 2$ then $\phi(n)$ is an ——— integer.

5. If a, b, c are integers and $g.c.d.(a, b, c) = 1$ then $gcd\,(a, b) =$ ———.

6. If N is a positive integer then $\sum_{n=1}^{N} \sigma(n) = \sum_{n=1}^{N}$ ———.

7. The value of $\sum_{n=1}^{6} T(n)$ is ———.

8. $a^p \equiv a \pmod p$ for any integer a if p is a ———.

9. The linear congruence $ax \equiv b \pmod n$ has a unique solution modulo n if $g.c.d.\ (a, n) =$ ———.

10. If $ca \equiv cb \pmod n$ and $g.c.d.(c, n) = 1$ then $a \equiv$ ——— (mod n).

11. When two integers a and b are said to be relatively prime.

12. Find the lcm (143, 227).

($12 \times \frac{1}{4} = 3$ weightage)

Turn over

II. Short answer type questions. (Answer all *nine* questions)

13. Prove that if $gcd\ (a, b) = d$ then $g.c.d. \left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

14. Find the remainder when $1! + 2! + 3! + \ldots\ldots + 99! + 100!$ is divided by 12.

15. State Chinese Remainder theorem.

16. Find $\sigma(180)$.

17. Find $\phi(16)$.

18. Show that no skew–symmetric matrix can be of rank 1.

19. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$ then find $\rho(A)$.

20. Define Null space of a matrix.

21. Examine whether the system of equations $4x + 6y = 5, 6x + 9y = 7$ has a solution.

(9 × 1 = 9 weight

III. Answer any *five* questions from seven :

22. Show that every square is of the form $3m$ or $3m + 1$.

23. Prove that the product of any three consecutive integers is divisible by 6.

24. Prove that $\sqrt{2}$ is irrational.

25. Use the binary exponentiation algorithm to compute 5^{110} (mod 131).

26. Reducing to the normal form find the rank of the matrix $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$.

27. If A is a non–singular matrix prove that the eigenvalues of A^{-1} are the reciprocals of eigenvalues of A.

28. Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

(5 × 2 = 10 weigh

Answer *two* questions from three.

29. State and prove Wilson's theorem.

30. Prove that $3^{2n+1} + 2^{n+2}$ is divisible by 7.

31. Find the characteristic roots and the corresponding characteristic vectors of the matrix $\begin{bmatrix} -2 & -1 \\ 5 & 4 \end{bmatrix}$.

(2 × 4 = 8 weightage)