

## SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 12—NUMBER THEORY AND LINEAR ALGEBRA

Time : Three Hours

Maximum : 30 Weightage

I. Answer all *twelve* questions :

- 1 If  $\text{Rank}(A) = \text{Ran}(c) < n$  then the system :
  - (a) Is consistent.
  - (b) Has infinite solutions.
  - (c) Is inconsistent.
  - (d) None of these.
- 2 If  $A$  is a non-singular matrix then  $(A^{-1})^{-1} = \text{_____}$ .
- 3 If  $n > 1$  the sum of the positive integers less than  $n$  and relatively prime to  $n$  is \_\_\_\_\_.
- 4 Define the rank of a matrix.
- 5 The value of  $\sum_{n=1}^6 \left[ \frac{6}{n} \right]$  is \_\_\_\_\_.
- 6 Find  $\phi(9)$ .
- 7 The value of  $\sigma(12) = \text{_____}$ .
- 8 If  $ca \equiv cb \pmod{p}$  and  $p \nmid c$ , where  $p$  is a prime number then \_\_\_\_\_  $\equiv b \pmod{p}$ .
- 9 If  $p, q_1, q_2, \dots, q_n$  are all primes and  $p \mid q_1 q_2 \dots q_n$  then prove that  $p = q_k$  for some  $k$ .
- 10 The linear congruence  $ax \equiv b \pmod{n}$  has a unique solution modulo  $n$  if  $\text{gcd}(a, n) = \text{_____}$ .
- 11 State Wilson's theorem.
- 12 If  $n$  is a positive integer then :
 
$$\sum_{n=1}^N \mathcal{T}(n) = \sum_{n=1}^N \text{_____}$$

(12  $\times$   $\frac{1}{4}$  = 3 weightage)II. Answer all *nine* questions :

- 13 State Cayley-Hamilton theorem.
- 14 Define Nullity of a matrix.

Turn over

15 What is the value of  $\begin{bmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix}$ .

16 If  $A = \begin{bmatrix} 1 & 5 & 2 \\ 3 & 2 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 2 & 1 \end{bmatrix}$  find  $\rho(A B)$ .

17 Under what condition the rank of the matrix  $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$  is 3.

18 Given integers  $a, b, c$ . If  $\gcd(a, b) = 1$  and  $\gcd(a, c) = 1$  what is  $\gcd(a, bc)$ .

19 Prove that  $\sigma(mn) = \sigma(m)\sigma(n)$ .

20 If  $ca \equiv cb \pmod{n}$  then prove that  $a \equiv b \pmod{n/d}$  where  $d = \gcd(c, n)$ .

21 If  $p$  is a prime and  $p \nmid ab$  then prove that  $p \nmid a$  or  $p \nmid b$ .

(9 × 1 = 9 we)

III. Answer any five questions from seven :

22 Prove that the cube of any integer has one of the forms  $7k$  or  $7k \pm 1$ .

23 Find the g.c.d. of (595, 252).

24 Prove that there is an infinite number of primes.

25 Prove that 13 divides  $4^{2n+1} + 3^{n+2}$ .

26 Prove that if  $n > 1$  the sum of the positive integers less than  $n$  and relatively prime

$$\frac{1}{2} n \phi(n).$$

27 Reducing to the normal form find the rank of the matrix  $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ .

28 Find the inverse of  $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 1 & 0 \end{bmatrix}$ .

(5 × 2 = 10 weig

IV. Answer any *two* questions from three :

29 Find non-singular matrices  $P$  and  $Q$  such that  $PAQ$  is in the normal form

$$\text{where } A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix}.$$

30 State and prove Fermat's theorem.

31 Use the Euclidean Algorithm to obtain integers  $x$  and  $y$  satisfying :

$$\gcd(858, 325) = 858x + 325y$$

(2 × 4 = 8 weightage)