

C 60049

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Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2019

(CUCBCSS)

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all the twelve questions.

Each question carries 1 mark.

1. Solve for z : $5z = 2iz$.
2. State Cauchy-Goursat theorem with full assumptions involved.
3. Verify whether $f(z) = \bar{z}$ is analytic or not at $z = 0$.
4. Find the simple poles, if any for the function $f(z) = \frac{(z-2)^2}{z^3(z^2+1)}$.
5. Is $u(x, y) = x^2 + y^2 - xy$ a harmonic function? Justify your claim.
6. Define a simply connected domain.
7. Fill in the blanks : The real part of $\cosh(2z)$ is _____.
8. Fill in the blanks : The locus of the points z satisfying $|z - 2i| = 2|i - 1|$ is a/an _____.
9. If an infinite series of complex numbers converges, then show that its n^{th} term converges to zero.
10. If R is the radius of convergence of $\sum a_n z^n$, find the radius of convergence of $\sum n^2 a_n z^n$.
11. What do you mean by a contour?
12. Find i^i .

(12 × 1 = 12 marks)

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Section B

Answer any **ten** out of fourteen questions.
Each question carries 4 marks.

13. Find the real and imaginary parts of the function $f(z) = \log(z)$.
14. Verify Cauchy-Riemann equations for the function $f(z) = z^3$.
15. Show that $\tan^{-1}(z) = \frac{i}{2} \log \frac{i+z}{i-z}$.
16. Show that the zeros of an analytic function are isolated.
17. Evaluate the line integral of $f(z) = z^2$ over the line joining i to $2i - 1$.
18. Find the radius of convergence of the power series: $\sum_{n=0}^{\infty} \frac{n! z^n}{n^n}$.
19. Verify Cauchy-Goursat theorem for $f(z) = z^2$ when the contour of integration is the circle with centre at origin and radius 3 units.
20. Locate the poles and zeros, if any, of $f(z) = \cos(1/z)$ in the complex plane.
21. Find all the solutions of $e^z = 3$.
- Find the residue of $f(z) = \sin(z)/z^2$ at $z = 0$ and evaluate the integral of $f(z)$ around the ellipse containing zero inside it.
- Using the definition of continuity show that the composite of two continuous functions is continuous.
- Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.
- Which one is bigger: $\|z_1| - |z_2\|$ or $|z_1 - z_2|$. Prove your claim.
- Determine all the poles of the $f(z) = \sec^2 z$ lying in the disc $|z - \pi/2| \leq 2$.

(10 × 4 = 40 marks)

Section C

Answer any **six** out of nine questions.
Each question carries 7 marks.

27. Determine the nature of the singularities of the function $f(z) = \sin(1/z)$. Does this function have zeros? Find them if any.
28. Evaluate $\oint_C \frac{z}{(z-a)(z-b)}$ discussing the cases of containment of the points $a \neq 0$ and $b \neq 0$ inside and outside the simple closed curve C .
29. Find the Laurentz series expansion of $f(z) = \frac{z}{(z-1)^2(z-2)}$ discussing the various regions of validity for the expansion.
30. Prove the converse of Cauchy-Goursat's integral theorem by fully stating the assumptions involved.
31. Find the harmonic conjugate of $u(x, y) = e^x(x \cos y - y \sin x)$ and find the corresponding analytic function $f(z)$ for which $u(x, y) = \operatorname{Re}(f(z))$. Express the result for $f(z)$ in terms of z only.
32. Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin, even though Cauchy Riemann equations are satisfied at that point.
33. How do we convert the Cauchy-Riemann equation into the corresponding polar form? Prove the formulas for conversion in detail.
34. Show that the derived series has the same radius of convergence as the original series.
35. Determine the locus of points of z in the complex plane satisfying the equation $|z-1| + |z-2| = 3$.

(6 × 7 = 42 marks)

Section D

Answer any **two** out of three questions.
Each question carries 13 marks.

36. (a) State and prove fundamental theorem of Algebra.
- (b) Find the residues of $f(z) = \frac{z^2}{(z-1)^2(z-2)}$ at its poles.

Turn c

37. (a) State and prove Liouville's theorem.

(b) Prove or disprove: $|\sin(z)| \leq 1$ for all complex numbers z . Justify your claim.

38. (a) Evaluate using the method of residues: $\int_0^{2\pi} \frac{1}{a + b \cos \theta} d\theta, a > b > 0$.

(b) Evaluate $\int_0^{\infty} \frac{1}{x^4 + a^4} dx, a > 0$.

(2 × 13 = 26 marks)