25704 (Pages : 2) Name..................................

Reg. No..................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2012

(CCSS)

Mathematics—Core Course

MM 6B 09—REAL ANALYSIS

: Three Hours Maximum : 30 Weightage

I. Objective Type Questions (Answer all *twelve* questions) :

1 State True *or* False. "Continuous functions are always bounded".

2 Does the function $f(x) = x^3 - 4x - 3$ has a root in [2, 3] ?

3 Give an example to a function which is continuous but not uniformly continuous.

4 If I = [0 1], find the norm of the Partion P : = (0 0.2 0.6 0.8 0.9 1).

5 State True *or* False. "An unbounded function cannot be Riemann integrable".

6 Consider the Signum function on [–10, 10]. Then the value of $\int_{-10}^{+10} \text{Sgn}(x)dx$ is ———.

7 Value of $\lim(x^n)$ for $x \in (-1, 1)$ is ———.

8 State Weierstrass–M test.

9 Define uniform norm of a bounded function ϕ on $A \subseteq R$ and $\phi : A \to R$.

10 Give an example of an improper integral.

11 Examine the convergence of $\int_0^\infty \frac{1}{x^2} dx$.

12 Write the relation between Beta and Gamma functions.

(12 × ¼ = 3 weightage)

II. Short Answer Type Questions. (Answer all *nine* questions) :

13 Define a bounded function. Also give an example to a function which is not bounded.

14 Find the absolute maximum and absolute minimum of $g(x) = x^2$ in A : = [–1, 1].

15 State intermediate value theorem.

16 Show that a constant function on [a b] is Riemann integrable on [a b].

17 State fundamental theorem of calculus.

18 Show that $g_n(x) = x^n$ for $x \in [0, 1]$; $n \in N$ is not uniformly convergent.

Turn over

19 Test for uniform convergence

$$f_n(x) = \frac{nx}{1+n^2x^2}, \text{ for all real } x.$$

20 Examine the convergence of $\int_1^\infty \frac{dx}{x^3}$.

21 Show that $\beta(m, n) = \beta(n, m)$.

(9 × 1 = 9 w

III. Short Essay Questions. (Answer any *five* from seven questions) :

22 Let I : $[a, b]$. Let f : I → R be a continuous function, prove that f is bounded on I.

23 Let I be an interval and f : I → R be continuous on I. Prove that f (I) is an interval i

24 If $f \in R[a, b]$, prove that the value of the integral is uniquely determined.

25 Prove that every continuous function is integrable.

26 Prove that A sequence of functions $\{f_n\}$ which is bounded on $A \subseteq R$ converges un to f; iff $\|f_n - f\|_A \to 0$.

27 Show that $\int_\pi^\infty \frac{\sin x}{x} dx$ converges conditionally.

28 Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, m)$.

(5 × 2 = 10 wei

IV. Essay Questions (Answer any *two* from three questions) :

29 Let I = $[a, b]$ and f : I → R be continuous if $f(a) < 0 < f(b)$ or if $f(a) > 0 > f(b)$ prove th exists c in (a, b) with $f(c) = 0$.

30 (i) If f and g are in R $[a, b]$, prove that $f + g$ is also in R $[a, b]$.

(ii) State Cauchy criterion for integrability of a function and use if to show that the D

function $f(x) = \begin{cases} 1 \text{ when } x \text{ is rational} \\ 0 \text{ when } x \text{ is irrational} \end{cases}$ is not Riemann integrable in [0 1].

31 (i) State and prove Weierstrass M-test.

(ii) Prove that $\Gamma n = (n-1)\Gamma(n-1)$

(2 × 4 = 8 wei