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(Pages : 3)

Name.....

Reg. No.....

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 09—REAL ANALYSIS

Three Hours

Maximum : 30 Weightage

Part A

Answer all *twelve* questions :

- 1 State True or False " $f(x) = \frac{1}{x}$ has neither an absolute maximum nor an absolute minimum on the set $(0, \infty)$ ".
- 2 Does $4 \sin x = x$ has a positive solution in $\left(\frac{\pi}{2}, \pi\right)$.
- 3 Give an example to a uniformly continuous function on $[0, b]$; $b > 0$.
- 4 Find the norm of the partition $p := (0, 2, 3, 4)$.
- 5 Give an example to function which is Riemann integrable in $[a, b]$.
- 6 State True or False "Every continuous function is integrable".
- 7 Define point-wise convergence of a sequence of function $\{f_n(n)\}$.
- 8 Show that $f_n(x) = \frac{1}{x+n}$ in uniformly convergent in $[0, b]$; $b > 0$.
- 9 Define uniform norm of a bounded function ϕ on $A \subseteq \mathbb{R}$ and $\phi : A \rightarrow \mathbb{R}$.
- 10 Define an improper integral.
- 11 Define Beta function.
- 12 If n is a positive integer value of $\sqrt{n+1}$ is _____.

(12 × ¼ = 3 weightage)

Turn over

Part B

Answer all **nine** questions.

- 13 Define absolute maximum and absolute minimum of $f : A \rightarrow \mathbb{R} ; A \subseteq \mathbb{R}$.
- 14 State maximum–minimum theorem.
- 15 Show that the function $f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$ is not integrable on any $[a, b]$.
- 16 If $f(x) \leq g(x) \forall x \in [a, b]$, then show that $\int_a^b f(x) dx \leq \int_a^b g(x) dx$.
- 17 Use fundamental theorem of calculus to evaluate $\int_a^b x dx$.
- 18 Show that $G(x) = x^n(1-x)$ for $x \in A := [0, 1]$ converges uniformly to $g(x) = 0$.
- 19 Show that $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ is uniformly convergent in $[-1, 1]$.
- 20 Examine the convergence of $\int_1^{\infty} \frac{dx}{\sqrt{x}}$.
- 21 Express $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} dx$ as a Beta function.

(9 × 1 = 9)

Part C

Answer any **five** questions from 7.

- 22 State and prove Intermediate value theorem.
- 23 Define a Lipschitz function. Also prove that a Lipschitz function $f : A \rightarrow \mathbb{R}$ is continuous on A .

- 24 Show that $f(x) = x^2$ is Riemann integrable on $[0, k]$.
- 25 If $f : [a, b] \rightarrow \mathbb{R}$ is monotonic on $[a, b]$ then prove that $f \in \mathcal{R}[a, b]$.
- 26 State and prove Weierstrass M-test.
- 27 If $\int_a^\infty |f(x)| dx$ converges, show that $\int_a^\infty f(x) dx$ converges.
- 28 Using Beta functions prove that :

$$\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$$

(5 × 2 = 10 weightage)

Part D

Answer any two from four questions.

- 29 (a) Let $I := [a, b]$ be a closed bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . Prove that f is bounded on I .
- (b) Define a step function and give an example.
- 30 Prove that if f is Riemann integrable on $[a, b]$ then it is bounded on $[a, b]$.
- 31 (a) Test for uniform convergence $\sum_1^\infty \frac{\sin nx}{n^p}$ for $p > 1$.
- (b) Prove that $\int_0^\infty e^{-ax} x^{n-1} dx = \frac{\sqrt{n}}{a^n}$; where $a > 0, n > 0$.

(2 × 4 = 8 weightage)