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Name.....................................

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SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2012

(CCSS)

Mathematics—Core Course

MM 6B 10—COMPLEX ANALYSIS

: Three Hours Maximum : 30 Weightage

Section A

Answer all **twelve** *questions.*

1. What is the real part of z^2 ?
2. The real and imaginary parts of an analytic function are ——— functions.
3. Give any singular point of the function $\frac{2z+1}{z\left(z^2+1\right)}$.
4. Choose the correct answer :

   $|e^z| =$ ———.

   (a) e^x. (b) e.

   (c) e^{-x}. (d) e^2.
5. The period of e^z is ———.

   (a) 2π. (b) $2\pi i$.

   (c) π. (d) πi.
6. Express $\sin x$ in terms of e^{ix}.
7. What is the parametric form of the unit cicle ?
8. Every bounded entire function is ———.
9. The region of convergence of $1 + z + z^2 + \ldots$ is ———.
10. $\int_{|z|=1} \left(z^2+1\right) dz =$ ———.
11. For the function $f(z) = \frac{\sin z}{z}$, $r = 0$ is :

    (a) Pole of order 1. (b) Removable singular point.

    (c) Essential singular point. (d) Pole of order 2.
12. Identify a singular point of $\frac{1}{z+z^2}$.

(12 × ¼ = 3 weightage)

Turn over

Section B

Answer all **nine** *questions.*

13. If $f'(z)=0$ everywhere in a domain D, prove that $f(z)$ is a constant throughout D.

14. Define harmonic function and give example.

15. Show that $\log(-ei)=1-\frac{\pi}{2}i$.

16. Find the principal value of $(-i)^i$.

17. State Cauchy-Goursat Theorem.

18. Evaluate $\int_C \frac{dz}{z-a}$, where C is $|z-a|=R$.

19. State Taylor's theorem

20. Discuss the nature of singularity of $e^{1/z}$ at $z=0$.

21. For the function $f(z)=\frac{1-e^{2z}}{z^4}$, determine the order of the pole at $z=0$ and the corresp residue.

(9 × 1 = 9 weig

Section C

Answer any **five** *questions.*

22. Derive the Cauchy-Riemann equations of an analytic function.

23. Show that $u(x,y)=\sinh x\sin y$ is harmonic in a domain and find a harmonic conjugate of u.

24. Find all roots of the equations.

    (a) $e^z=-2$.

    (b) $\sinh z=i$.

25. State and prove fundamental theorem of algebra.

26. Evaluate $\int_C \frac{\exp(2z)}{z^4}dz$, where C is the circle $|z|=1$.

27. Obtain the Taylor series expansion of e^z about $z=1$ and state the region of validity.

28. Evaluate $\int_0^\infty \frac{dx}{(x^2+1)^2}$.

(5 × 2 = 10 weig

Section D

Answer any **two** *questions.*

State the prove Cauchy's integral formula.

Give two Laurent series expansions in powers of z for the function $f(z)=\dfrac{1}{z^2(1-z)}$ and specify the regions in which the expansions are valid.

Using residues, evaluate

$$\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}.$$

(2 × 4 = 8 weightage)