

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 10—COMPLEX ANALYSIS

Time : Three Hours

Maximum : 30 Weightage

Section A

Answer all the questions.

1. What is the value of :

$$\lim_{z \rightarrow \infty} \left(\frac{2z+i}{z+1} \right)$$

2. Find the imaginary part of
- $z + i$
- .

- 3.
- $f(z) = u(x, y) + iV(x, y)$
- is analytic in a domain D if and only, V is _____ of u.

4. What is the real part of
- e^z
- ?

5. What is the period of
- $\sin z$
- ?

6. Express
- $\cos x$
- in terms of
- e^{ix}
- .

7. The value of
- $e^{i\pi}$
- is :

(a) 1.

(b) e .(c) -1 .

(d) 0.

8. The value of
- $\int_{|z|=1} \frac{dz}{z-4}$
- is _____.

(a) $2\pi i$.

(b) 0.

(c) 2π .

(d) 4.

9. The region of convergence of the series :

$$1 + \frac{z}{1!} + \frac{z^2}{2!} + \dots + \frac{z^n}{n!} + \dots \text{ is } \underline{\hspace{2cm}}$$

10. What is the sum function of the series
- $1 + z + z^2 + \dots + z^{n-1} + \dots$

$$\text{For } f(z) = \frac{z^2 - 4}{z - 2}, z = 2 \text{ is a } \underline{\hspace{2cm}}$$

(a) Removable singular point.

(b) Pole of order 1.

(c) Pole of order 2.

(d) Essential singular point.

Turn over

12. Identify the poles of:

$$\frac{2z}{z^2 - 1}$$

(12 × ¼ =

Section B

Answer all nine questions.

13. Show that $f'(z)$ does not exist at any point for $f(z) = 2x + ixy^2$.
14. If $f(z)$ and $\overline{f(z)}$ are both analytic in a domain D , prove that $f(z)$ is a constant.
15. Show that:

$$\text{Log}(1 - i) = \frac{1}{2} \ln 2 - \frac{\pi}{4} i.$$

16. Show that:

$$|\sinh z|^2 = \sinh^2 x + \sin^2 y.$$

17. State Cauchy's integral formula.
18. Evaluate:

$$\int_C \frac{dz}{z-1}, \text{ where } C \text{ is } |z-1| = 2.$$

19. Show that when $z \neq 0$

$$\frac{e^z}{z^2} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \dots$$

20. State Cauchy's residue theorem.

21. For the function $f(z) = \frac{1 - \cosh z}{z^3}$ determine the order of the pole at $z = 0$ and the corresponding residue.

(9 × 1 = 9

Section C

Answer any five questions.

22. Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic and find a harmonic conjugate $v(x, y)$.
23. If $f(z) = u(x, y) + iV(x, y)$ is analytic in a domain D , prove that u and V are harmonic.

24. Find the general solution of the equation :

$$\cosh z = \frac{1}{2}.$$

25. Evaluate $\int_C \frac{zdz}{(9-z^2)(z+i)}$, where C is the circle $|z| = 2$.

26. State and prove Liouville's theorem.

27. State and prove Taylor's theorem.

28. Evaluate $\int_0^x \frac{dx}{x^2+1}$.

(5 × 2 = 10 weightage)

Section D

Answer any two questions.

29. State and prove maximum modulus principle.

30. Give two Laurent series expansions in powers of z for the function :

$$f(z) = \frac{1}{z(1+z^2)}$$

the regions of validity of expansions.

31. Using residues, evaluate :

$$\int_{-\infty}^{\infty} \frac{\cos x \, dx}{(x^2+a^2)(x^2+b^2)} \quad (a > b > 0)$$

(2 × 4 = 8 weightage)