C 40401 (Pages : 4) Name.....................................

Reg. No.................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics—(Elective Course)

MM 6B 13 (E 02)—LINEAR PROGRAMMING AND GAME THEORY

(2009 Admissions)

Time : Three Hours Maximum : 30 Weightage

Part I

Answer **all** *questions.*

1. Maximum $Z = 2x_1 + 3x_2$ subject to $x_1 - x_2 \le 5,\ 3x_1 + 2x_2 \ge 1,\ x_1 \ge 0,\ x_2 \ge 0$ is a :
   (a) Linear programming problem.
   (b) Quadratic programming problem.
   (c) Transportation problem.
   (d) Assignment problem.
2. Define an objective function of a linear programming problem.
3. What is a slack variable ?
4. Which of the following is a convex set in R^2 ?
   (a) $\{(1, 0), (0, 1)\}$. (b) $\{(x, y) / y = \sin x\}$.
   (c) $\{(x, y) / x^2 + y^2 = 1\}$. (d) $\{(x, y) / a \le x \le b\}$.
5. Define a convex hull of a set in E^n.
6. Define a convex combination of the vectors $a_1, a_2, ...a_n$ in R^n.
7. Write an ortho normal basis of R^3.
8. Find a basic feasible solution of $x_1 + 2x_2 - x_3 + x_4 = 4,\ x_1 - x_2 + 2x_3 - x_4 = -2$ taking x_3 and x_4 as non-basic variables.
9. The set $\{(x, y) \in E^2 / 2x - y > 0\}$ is :
   (a) Closed and bounded. (b) Closed and convex.
   (c) Open and convex. (d) Compact and convex.
10. What is a zero-sum game ?

Turn over

11. Determine the dual of :

Minimize $Z = 4x_1 - x_2$

subject to

$$x_1 + x_2 \le 4,$$
$$2x_1 - x_2 \ge 3,$$
$$x_1 \ge 0,$$
$$x_2 \ge 0.$$

12. Define a translate of set S in a vector space.

($12 \times \frac{1}{4} = 3$ weightag

Part II

Answer **all** *questions.*

13. Show that (1, 1, 0), (0, 2, 1) and (1, –1, 2) is a linearly independent set.
14. Express (1, 3) as a linear combination of (1, 2) and (2, 3).
15. Find a feasible solution of the system :

$$x_1 + 2x_2 = 10, \quad x_1 + x_3 = 4$$
$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

16. Prove that intersection of two convex set is convex.
17. Rewrite the following L.P.P. in the standard form :

Maximize $Z = 3x - 2y$

subject to

$$x - y \le 1,$$
$$3x - 2y \le 6,$$
$$x \ge 0,$$
$$y \ge 0.$$

18. Obtain the dual of the L.P.P. :

Maximize $Z = 3x_1 + 4x_2$

subject to

$$x_1 - x_2 \le 1,$$
$$x_1 + x_2 \ge 4,$$
$$x_1 - 3x_2 \le 3, \text{ and}$$
$$x_1 \ge 0,$$
$$x_2 \ge 0.$$

19. State the fundamental theorem of Game theory.
20. Give an example of a balanced transportation problem.
21. Find a non-degenerate basic feasible solution of the system

$$x_1 + 2x_2 - x_3 + x_4 = 4$$
$$x_1 - x_2 + 2x_3 - x_4 = 2.$$

(9 × 1 = 9 weightage)

Part III

Answer any **five** *questions.*

22. Find graphically the feasible space of the following in equations :

$$x_1 + 2x_2 \leq 7,\ \ x_1 - x_2 \leq 4,\ \ x_1 \geq 0,\ \ x_2 \geq 0.$$

23. Show that the points (1, 2, 1) (2, 3, 0) and (1, 2, 2) form a basis for E^3.
24. Prove that convex hull of a set S in E^n consists of all convex combination of elements of S.
25. Find the convex hull of the set

$$S = \{(1, 2)\ (3, 7)\ (2, -1)\}.$$

26. Prove that if a constant is added to any row or column of the cost matrix of an assignment problem, an optimal solution of the original problem remains optimal for the new problem.
27. Using north-west corner rule find an initial solution to the transportation problem :

	D_1	D_2	D_3	
O_1	2	1	3	8
O_2	1	4	5	5
O_3	2	3	4	7
	6	9	5	

28. A company has 3 senior executives. Each is judged against each of the 3 positions and their rating are given by :

Position

Executives		I	II	III
	E_1	7	5	6
	E_2	8	4	7
	E_3	9	6	4

Assign each executive to one position so that sum of ratings for all 3 is highest.

(5 × 2 = 10 weightage)

Turn over

Part IV

Answer any **two** *questions.*

29. Solve by principle of duality :

Maximize $Z = 3x_1 - 2x_2$
subject to

$$x_1 \le 4, \quad x_2 \le 6\,,$$
$$x_1 + x_2 \le 5, \quad x_2 \ge 1$$
$$x_1 \ge 0, \quad x_2 \ge 0.$$

30. Find a basic solution by Vogel's approximation method :

	D_1	D_2	D_3	D_4	
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	90
	85	35	50	45	

31. Solve the game using principle of Dominance :

	Player B			
Player A	3	2	4	0
	3	4	2	4
	4	2	4	0
	0	4	0	8

(2 × 4 = 8 weightage)