C 40397 **(Pages : 3)** Name.....................................

Reg. No.....................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 12—NUMBER THEORY AND LINEAR ALGEBRA

Time : Three Hours Maximum : 30 Weightage

I. Answer all *twelve* questions :

1 If Rank (A) = Ran (c) < n then the system :

(a) Is consistent. (b) Has infinite solutions.

(c) Is inconsistent. (d) None of these.

2 If A is a non-singular matrix then $\left(A^{-1}\right)^{-1}$ = ———.

3 If $n > 1$ the sum of the positive integers less than n and relatively prime to n is ———.

4 Define the rank of a matrix.

5 The value of $\sum_{n=1}^{6} \left[\frac{6}{n}\right]$ is ———.

6 Find $\phi(9)$.

7 The value of $\sigma(12)$ = ———.

8 If $ca \equiv cb \pmod{p}$ and $p \times c$, where p is a prime number then ——— $\equiv b \pmod{p}$.

9 If $p, q_1, q_2, \ldots q_n$ are all primes and $p / q_1 q_2 \cdots q_n$ then prove that $p = q_k$ for same k.

10 The linear congruence $ax \equiv b (\bmod n)$ has a unique solution modulo n if gcd (a, n) = ———.

11 State Wilson's theorem.

12 If n is a positive integer then :

$$\sum_{n=1}^{N} \tau(n) = \sum_{n=1}^{N} \text{———}.$$

(12 × ¼ = 3 weightage)

II. Answer all *nine* questions :

13 State Cayley-Hamilton theorem.

14 Define Nullity of a matrix.

Turn over

15 What is the value of $\begin{bmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix}$.

16 If $A = \begin{bmatrix} 1 & 5 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 2 & 1 \end{bmatrix}$ find $\rho(AB)$.

17 Under what condition the rank of the matrix $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ is 3.

18 Given integers a, b, c . If g c d $(a, b) = 1$ and $g\ c\ d\ (a, c) = 1$ what is $g\ c\ d\ (a, bc)$.

19 Prove that $\sigma(mn) = \sigma(m)\,\sigma(n)$.

20 If $ca \equiv cb(\bmod n)$ then prove that $a \equiv b(\bmod n/d)$ where $d = \gcd(c,n)$.

21 If p is a prime and p / ab then prove that p/a or p/b.

(9 × 1 = 9 weightage)

III. Answer any *five* questions from seven :

22 Prove that the cube of any integer has one of the forms $7k$ or $7k \pm 1$.

23 Find the g.c.d. of (595, 252).

24 Prove that there is an infinite number of primes.

25 Prove that 13 divides $4^{2n+1} + 3^{n+2}$.

26 Prove that if $n > 1$ the sum of the positive integers less than n and relatively prime to n is $\frac{1}{2} n\,\phi(n)$.

27 Reducing to the normal form find the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$.

28 Find the inverse of $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 1 & 0 \end{bmatrix}$.

(5 × 2 = 10 weightage)

IV. Answer any *two* questions from three :

29 Find non-singular matrices P and Q such that PAQ is in the normal form where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix}$.

30 State and prove Fermat's theorem.

31 Use the Euclidean Algorithm to obtain integers x and y satisfying :

$$\gcd(858, 325) = 858\,x + 325\,y$$

(2 × 4 = 8 weightage)