C 40394 **(Pages : 3)** **Name..................................**

Reg. No..................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 09—REAL ANALYSIS

Time : Three Hours Maximum : 30 Weightage

Part A

Answer all *twelve* questions :

1 State True *or* False "$f(x) = \frac{1}{x}$ has neither an absolute maximum nor an absolute minimum on the set $(0, \infty)$".

2 Does $4 \sin x = x$ has a positive solution in $\left(\frac{\pi}{2}, \pi\right)$.

3 Give an example to a uniformly continuous function on $[0, b]$; $b > 0$.

4 Find the norm of the partition $p := (0, 2, 3, 4)$.

5 Give an example to function which is Riemann integrable in $[a, b]$.

6 State True *or* False "Every continuous function is integrable".

7 Define point-wise convergence of a sequence of function $\{f_n(n)\}$.

8 Show that $f_n(x) = \frac{1}{x+n}$ in uniformly convergent in $[0, b]$; $b > 0$.

9 Define uniform norm of a bounded function ϕ on $A \subseteq R$ and $\phi : A \to R$.

10 Define an improper integral.

11 Define Beta function.

12 If n is a positive integer value of $\sqrt{n+1}$ is ————————.

(12 × ¼ = 3 weightage)

Turn over

Part B

Answer all **nine** *questions.*

13 Define absolute maximum and absolute minimum of $f : A \to R ; A \subseteq R$.

14 State maximum–minimum theorem.

15 Show that the function $f(x) = \begin{cases} 0, \text{ when } x \text{ is rational} \\ 1, \text{ when } x \text{ is irrational} \end{cases}$ is not integrable on any interval.

16 If $f(x) \le g(x) \ \forall \ x \in [a, b]$, then show that $\int_a^b g\, dx \le \int_a^b g\, dx$.

17 Use fundamental theorem of calculus to evaluate $\int_a^b x\, dx$.

18 Show that $G(x) = x^n (1 - x)$ for $x\, t\, A := [0, 1]$ converges uniformly to $g(x) = 0$.

19 Show that $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ is uniformly convergent in [– 1 1].

20 Examine the convergence of $\int_1^\infty \frac{dx}{\sqrt{x}}$.

21 Express $\int_0^1 \frac{x^2}{\sqrt{1 - x^5}} dx$ as a Beta function.

(9 × 1 = 9 weightage)

Part C

Answer any **five** *questions from 7.*

22 State and prove Intermediate value theorem.

23 Define a Lipschitz function. Also prove that a Lipschitz function $f : A \to R$ is uniformly continuous on A.

24 Show that $f(n) = x^2$ is Riemann integrable on [0, k].

25 If $f:[a,b] \to R$ is monotonic on [a b] then prove that $f \in R[a,b]$.

26 State and prove Weierstrass M-test.

27 If $\int_a^\infty |f(x)|\, dx$ converges, show that $\int_a^\infty f(x)\, dx$ converges.

28 Using Beta functions prove that :

$$\int_0^1 \frac{x^2}{\sqrt{1-n^4}}\, dx \times \int_0^1 \frac{dx}{\sqrt{1+n^4}} = \frac{\pi}{4\sqrt{2}}.$$

(5 × 2 = 10 weightage)

Part D

Answer any **two** *from four questions.*

29 (a) Let I : = [a, b] be a closed bounded interval and let $f: I \to R$ be continuous on I. Prove that f is bounded on I.

(b) Define a step function and give an example.

30 Prove that if f is Riemann integrable on [a, b] then it is bounded on [a, b].

31 (a) Test for uniform convergence $\sum_1^\infty \frac{\sin nx}{n p}$ for $p > 1$.

(b) Prove that $\int_0^\infty e^{-ax} x^{n-1}\, dx = \frac{\sqrt{n}}{a^n}$; where $a > 0$, $n > 0$.

(2 × 4 = 8 weightage)